



Extraction of intensities from powder diffraction pattern

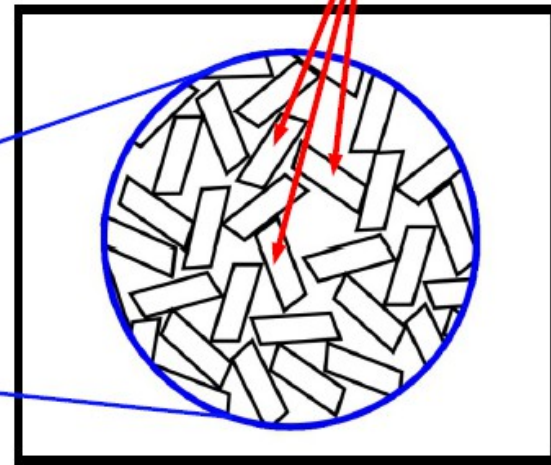
Important step before solving the crystal
structure



Sample

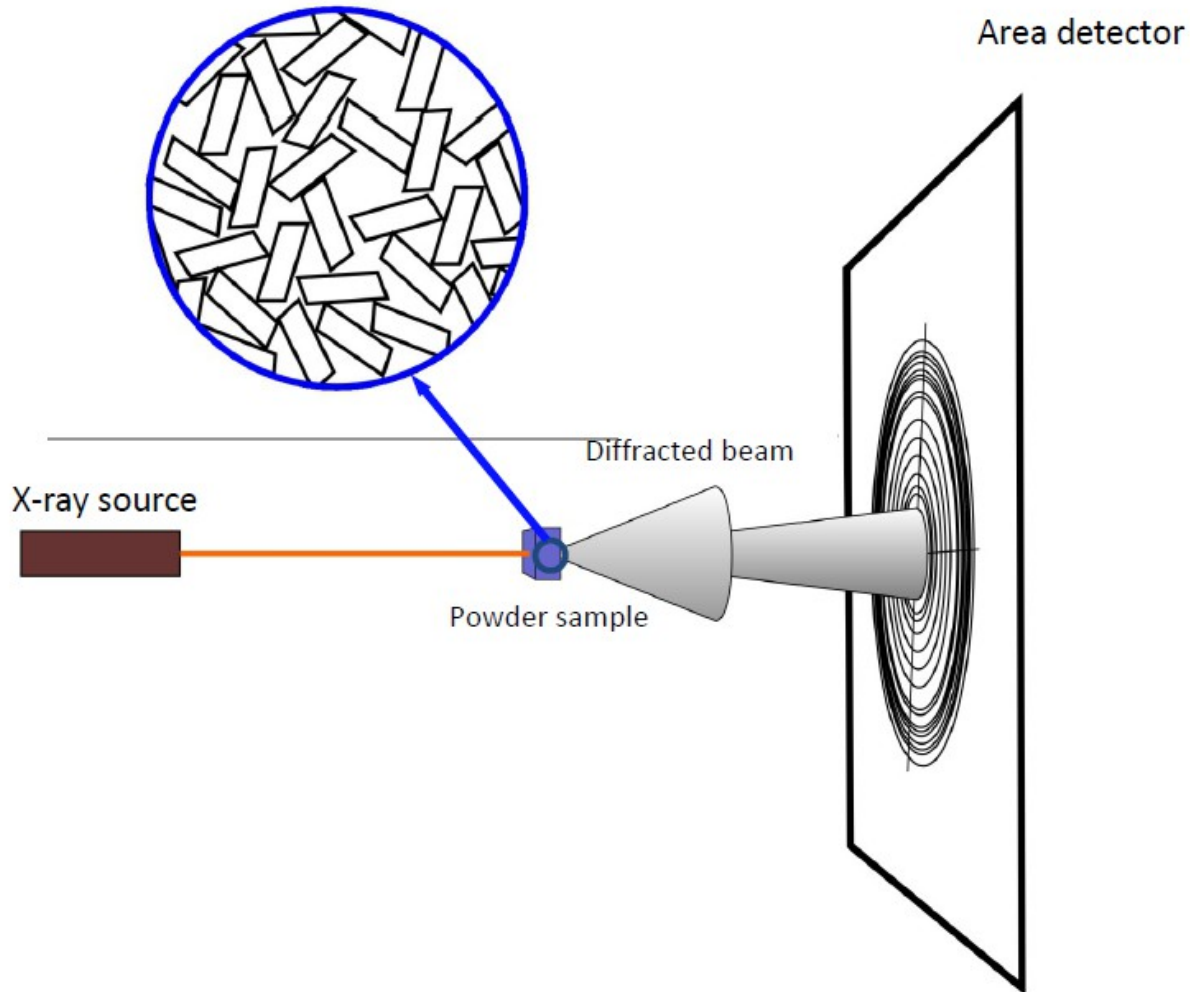
Powder sample = polycrystalline material

Size of crystals is around 10^{-6}m



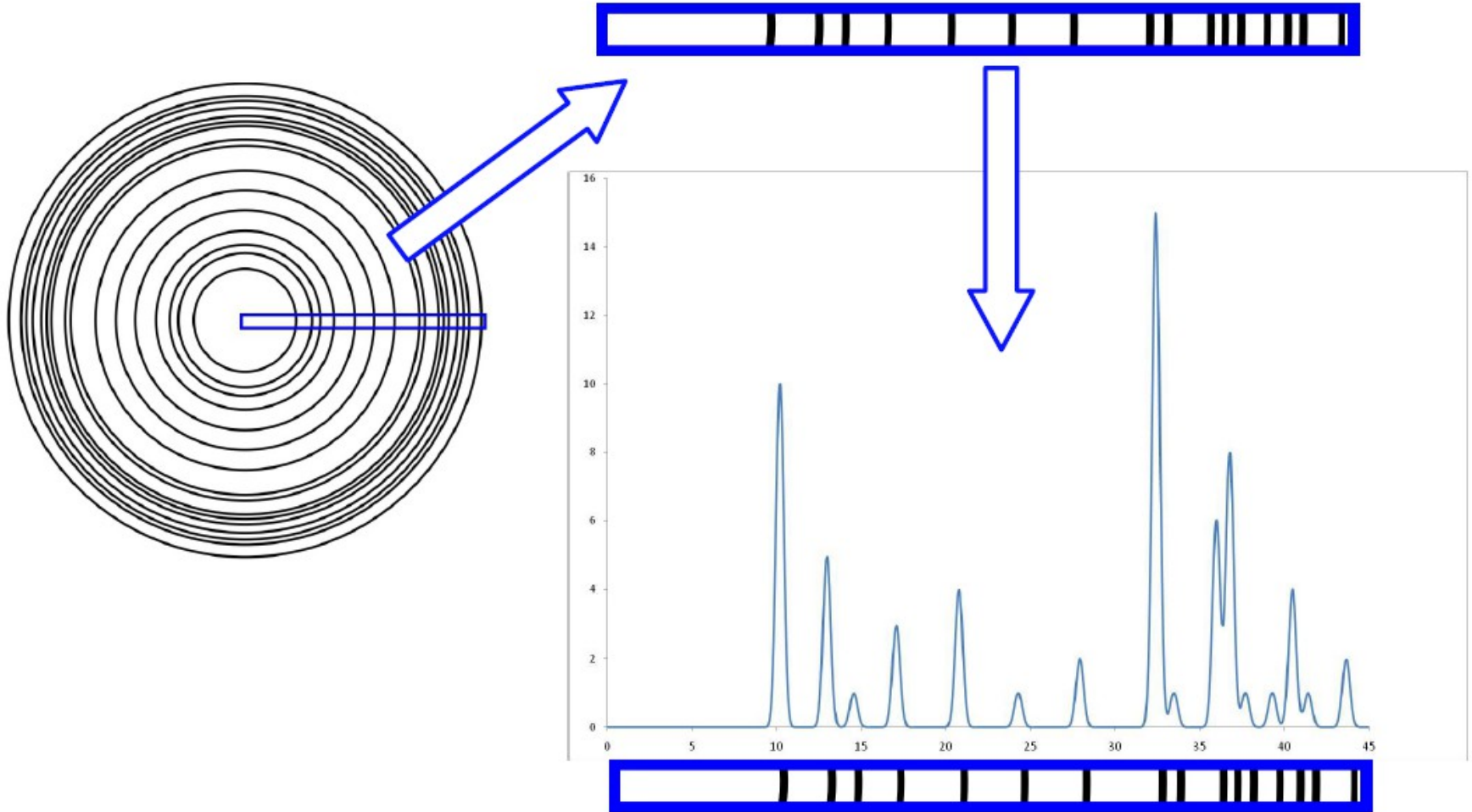
Small single-crystals

Diffraction of the powder sample

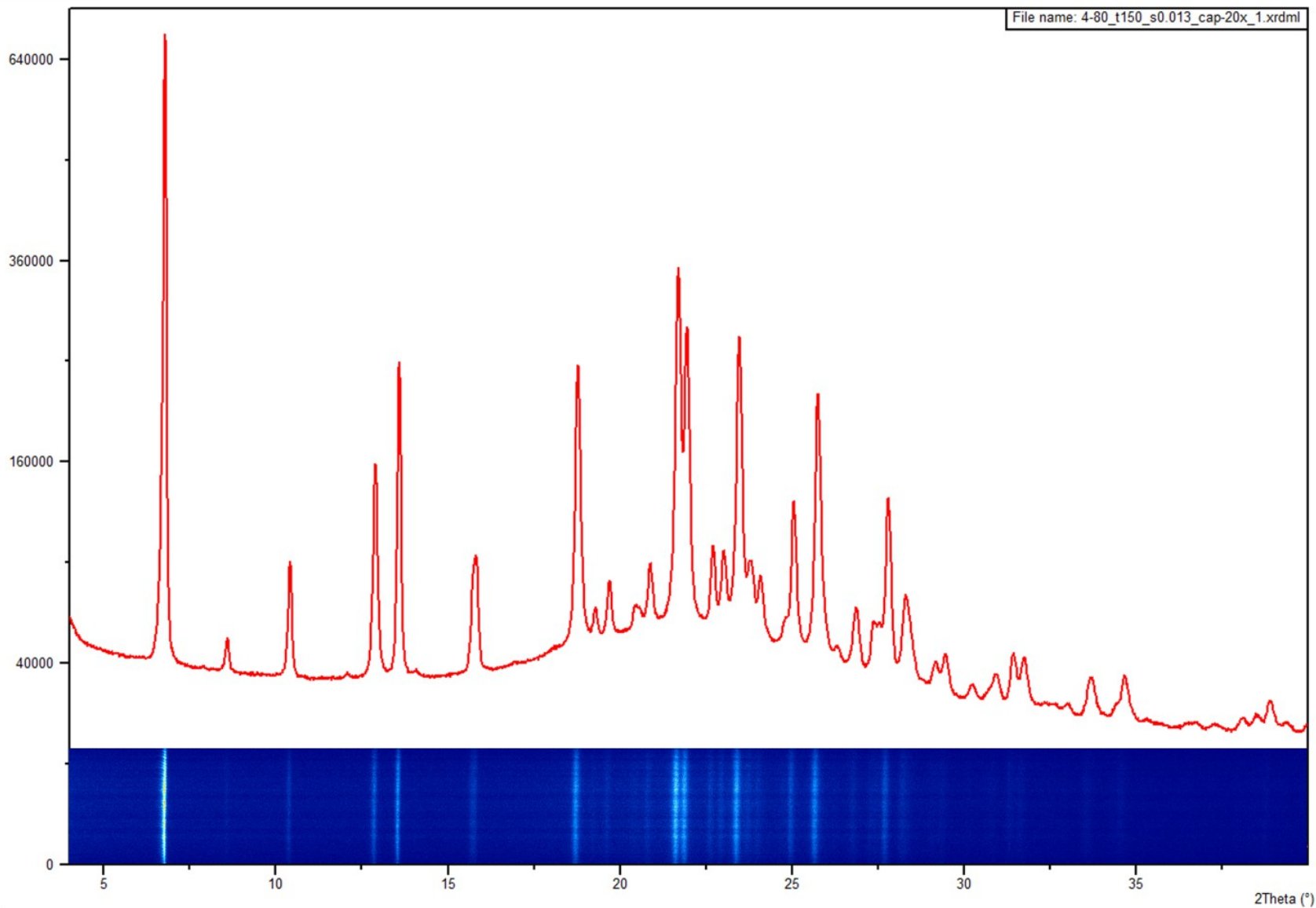




Diffraction of the powder sample



Diffraction of the powder sample

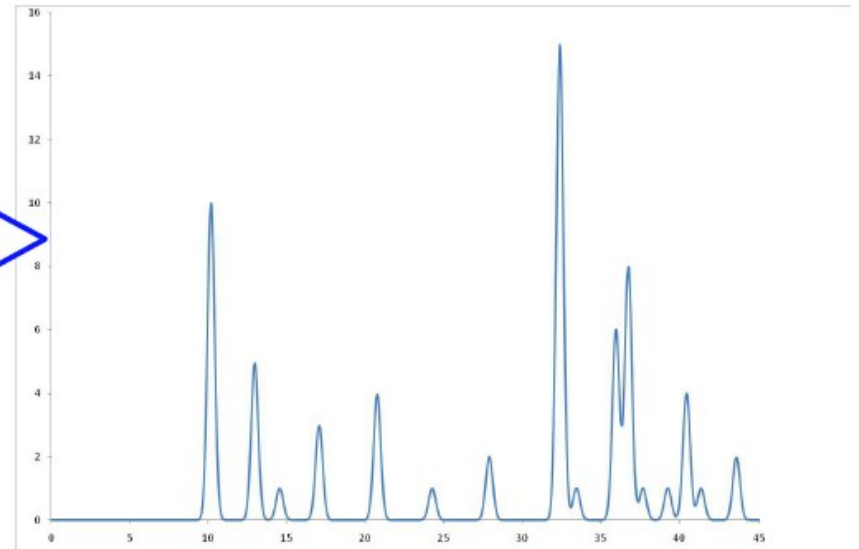
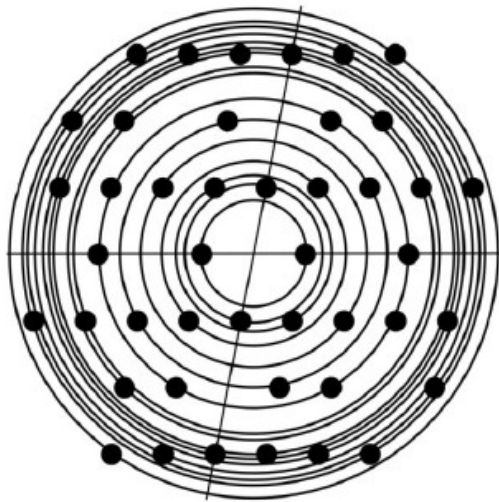


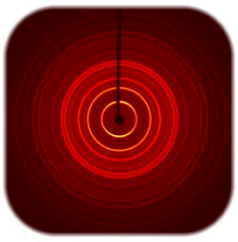


Diffraction of the powder sample

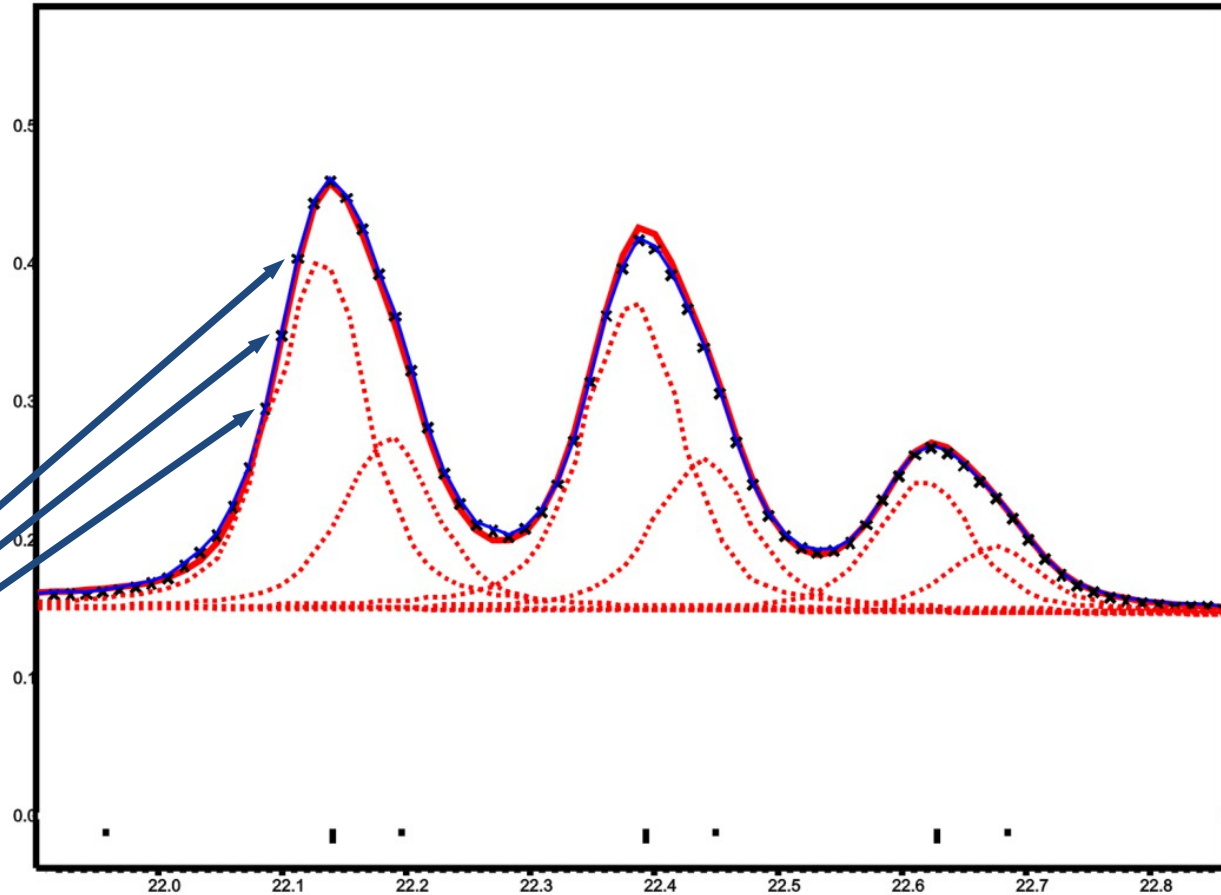
- **Information loss** – powder pattern is a 1D projection of 3D reciprocal space

How to describe peak shapes?
How to get intensities?





Powder pattern



Intenzita v každém bodě záznamu

$$y_{ci} = s \sum_h L_h |F_h|^2 \delta(2\theta_i - 2\theta_h) P_h A + y_{bi}$$

s – scale factor, L_h – Lorentz, polarization and multiplicity factor,

F_h – structure factor, δ – reflection profile, P_h – preferred orientation function, A – absorption factor, y_{bi} - background



Powder pattern

$$y_{ci} = s \sum_h L_h |F_h|^2 \delta(2\theta_i - 2\theta_h) P_h A + y_{bi}$$

$$|F_h|^2 \approx \mathbf{I}$$

$$G(b_G, x) = \frac{1}{\sqrt{2\pi b_G}} \exp(-x^2 / 2b_G^2)$$

$$L(b_L, x) = \frac{2}{\pi b_L} \frac{1}{1 + (2x/b_L)^2}$$

Unknown model – what is the intensity?

s – scale factor, L_h – Lorentz, polarization and multiplicity factor,
 F_h – structure factor, δ – reflection profile, P_h – preferred orientation function, A –
absorption factor, y_{bi} - background

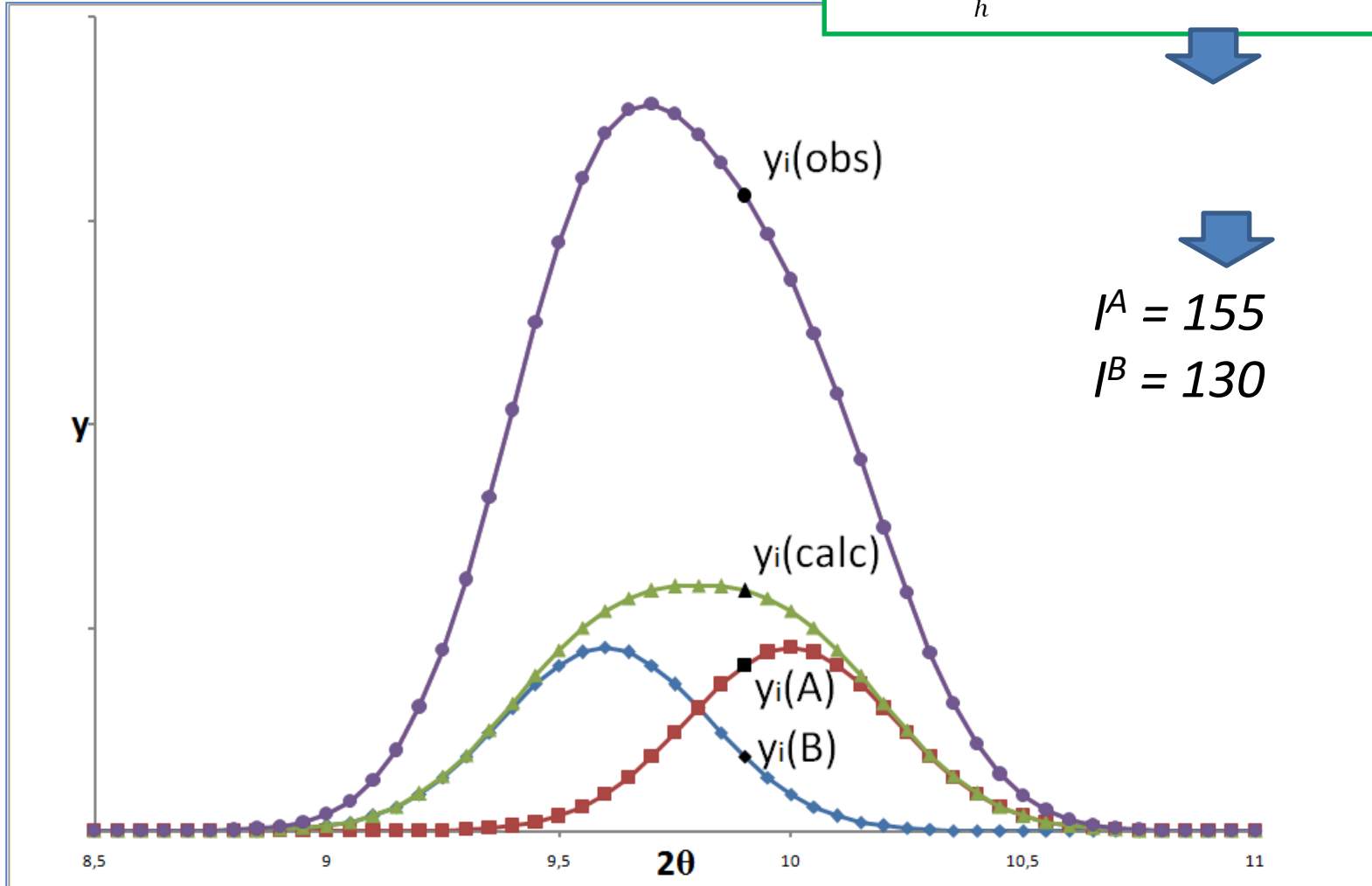


Le Bail method

$$I^A = 50$$

$$I^B = 50$$

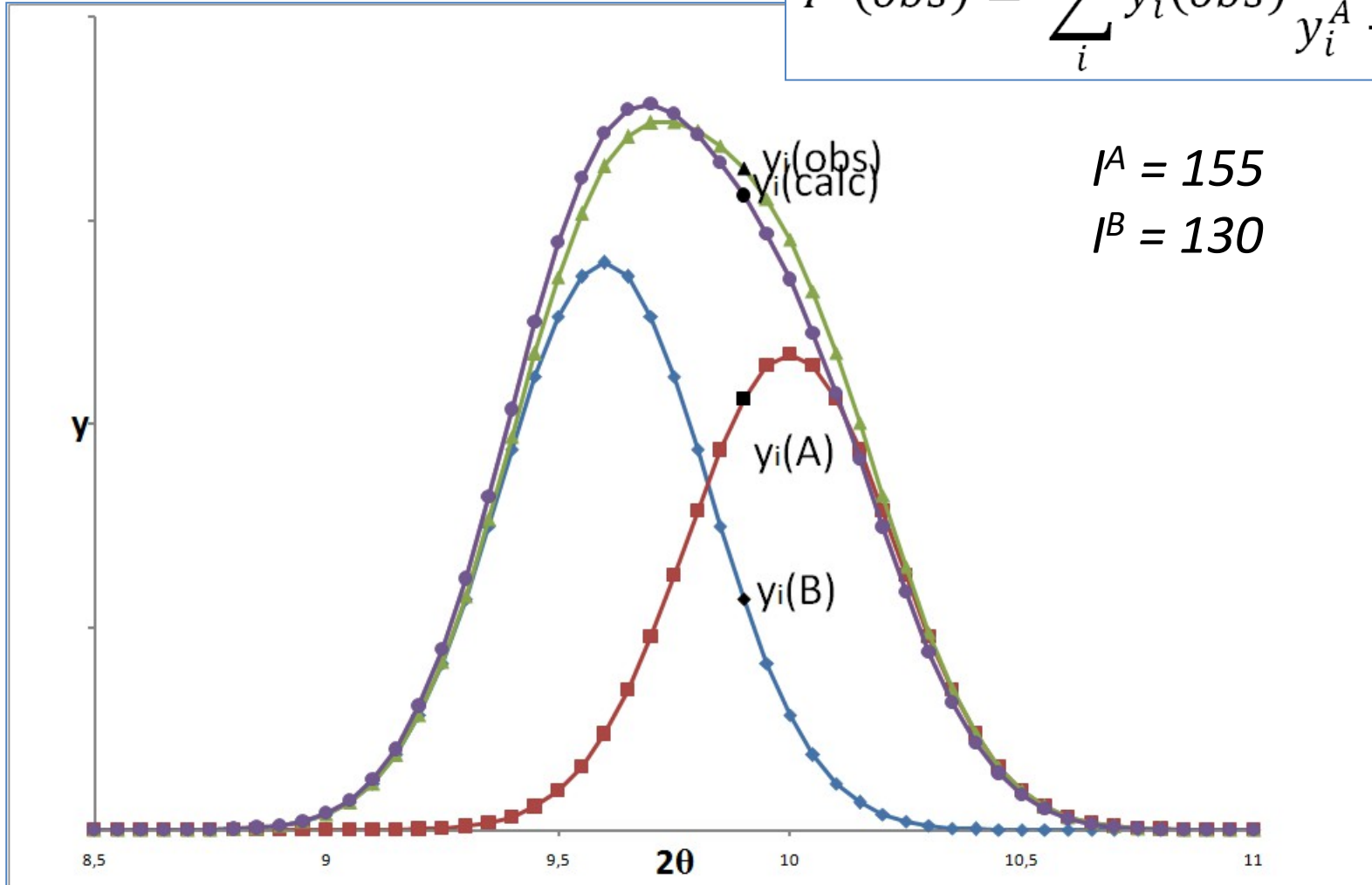
$$y_{ci} = s \sum_h L_h |F_h|^2 \delta(2\theta_i - 2\theta_h) P_h A + y_{bi}$$





Le Bail method

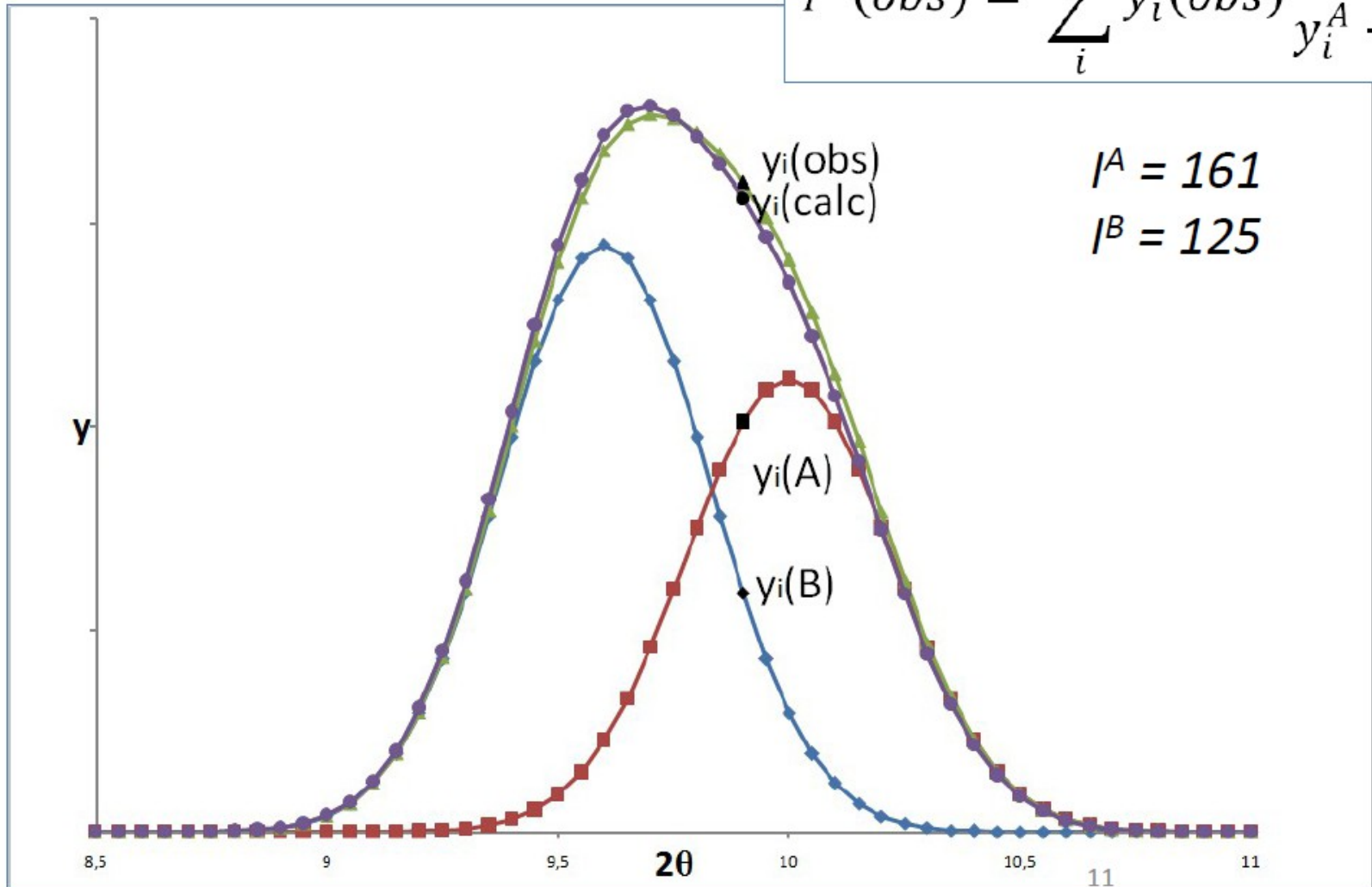
$$I^A(obs) = \sum_i y_i(obs) \frac{y_i^A}{y_i^A + y_i^B}$$



Le Bail method

$$I^A(obs) = \sum_i y_i(obs) \frac{y_i^A}{y_i^A + y_i^B}$$

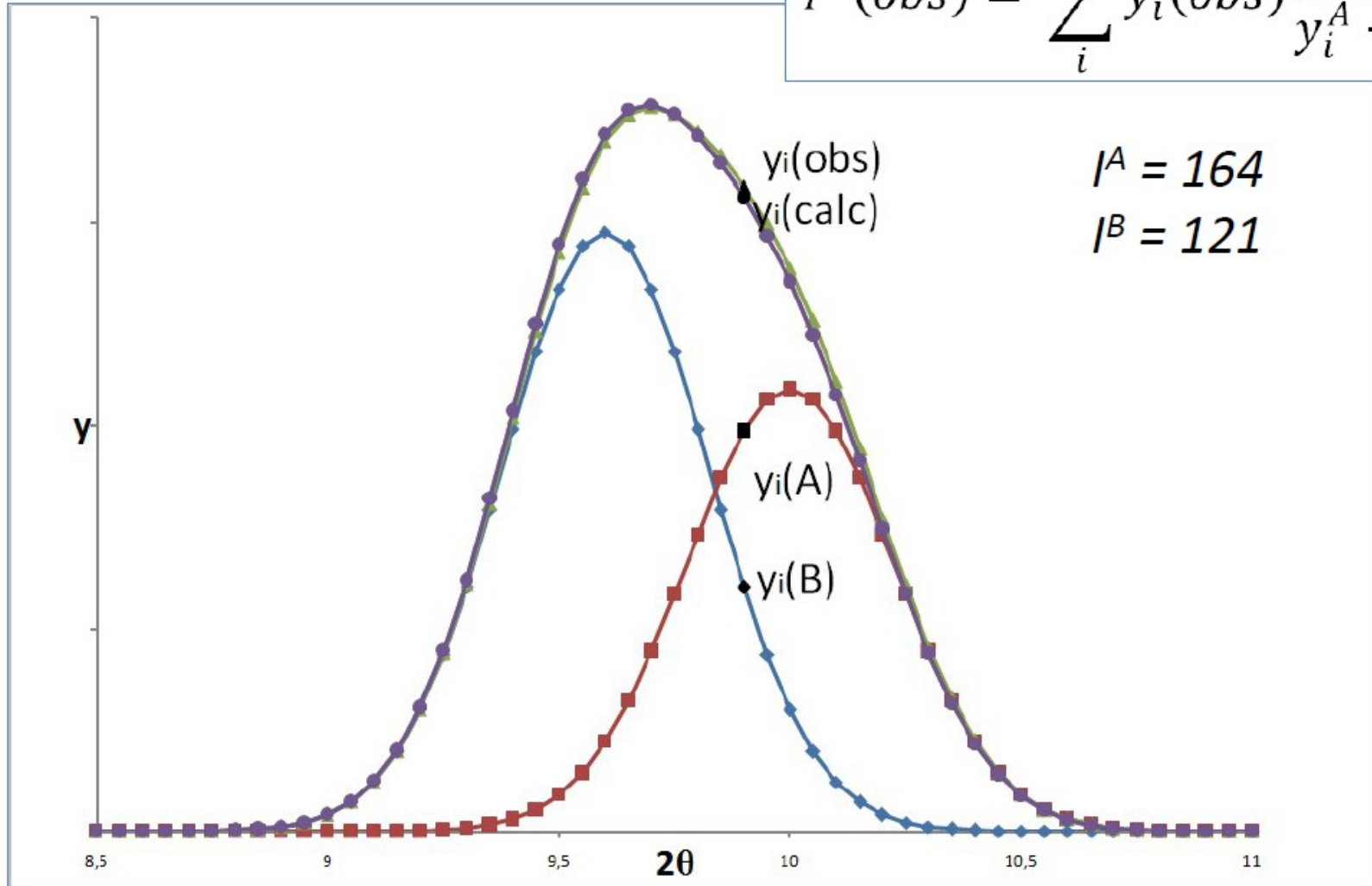
$$I^A = 161$$
$$I^B = 125$$





Le Bail method

$$I^A(obs) = \sum_i y_i(obs) \frac{y_i^A}{y_i^A + y_i^B}$$



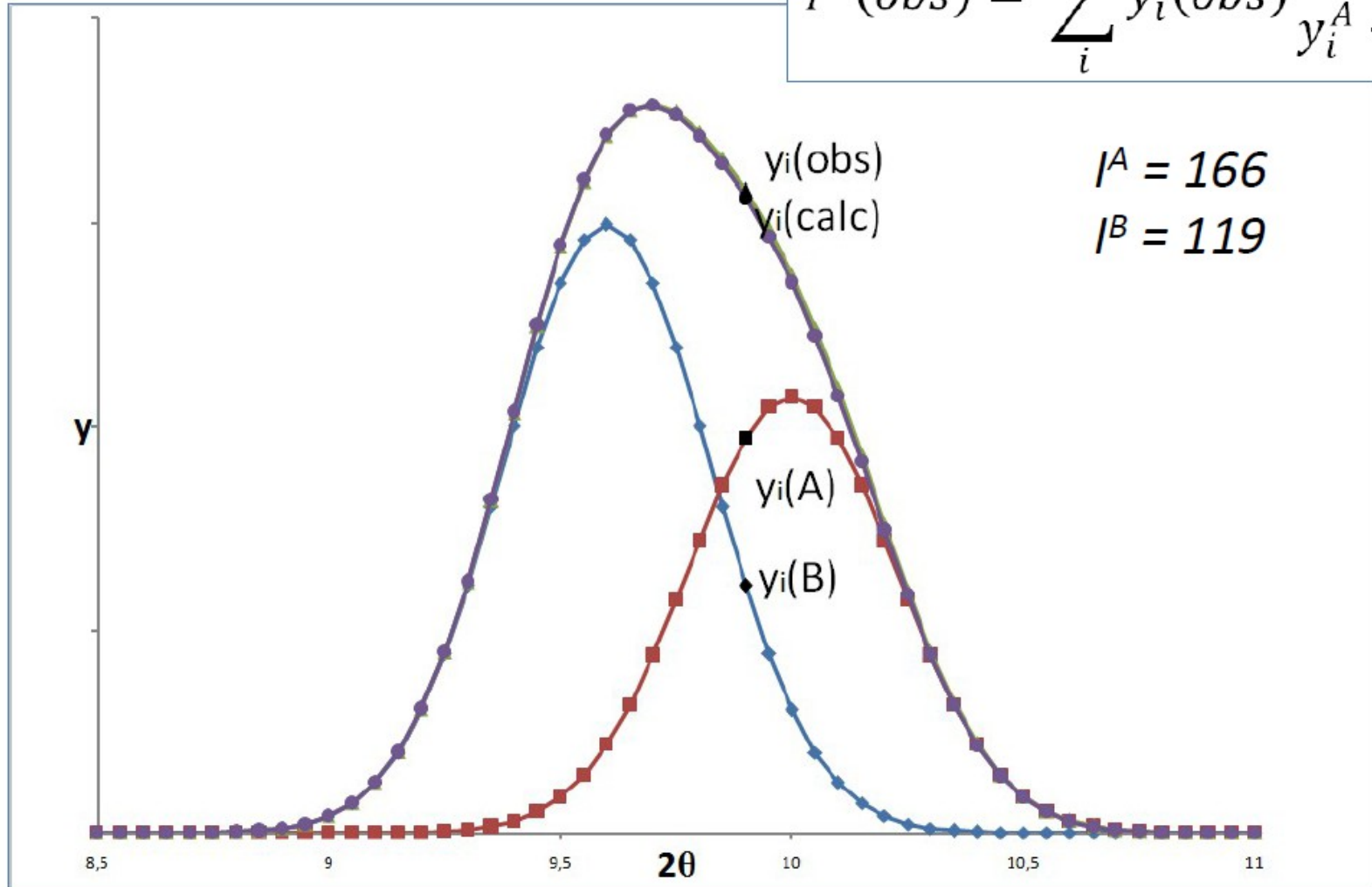


Le Bail method

$$I^A(obs) = \sum_i y_i(obs) \frac{y_i^A}{y_i^A + y_i^B}$$

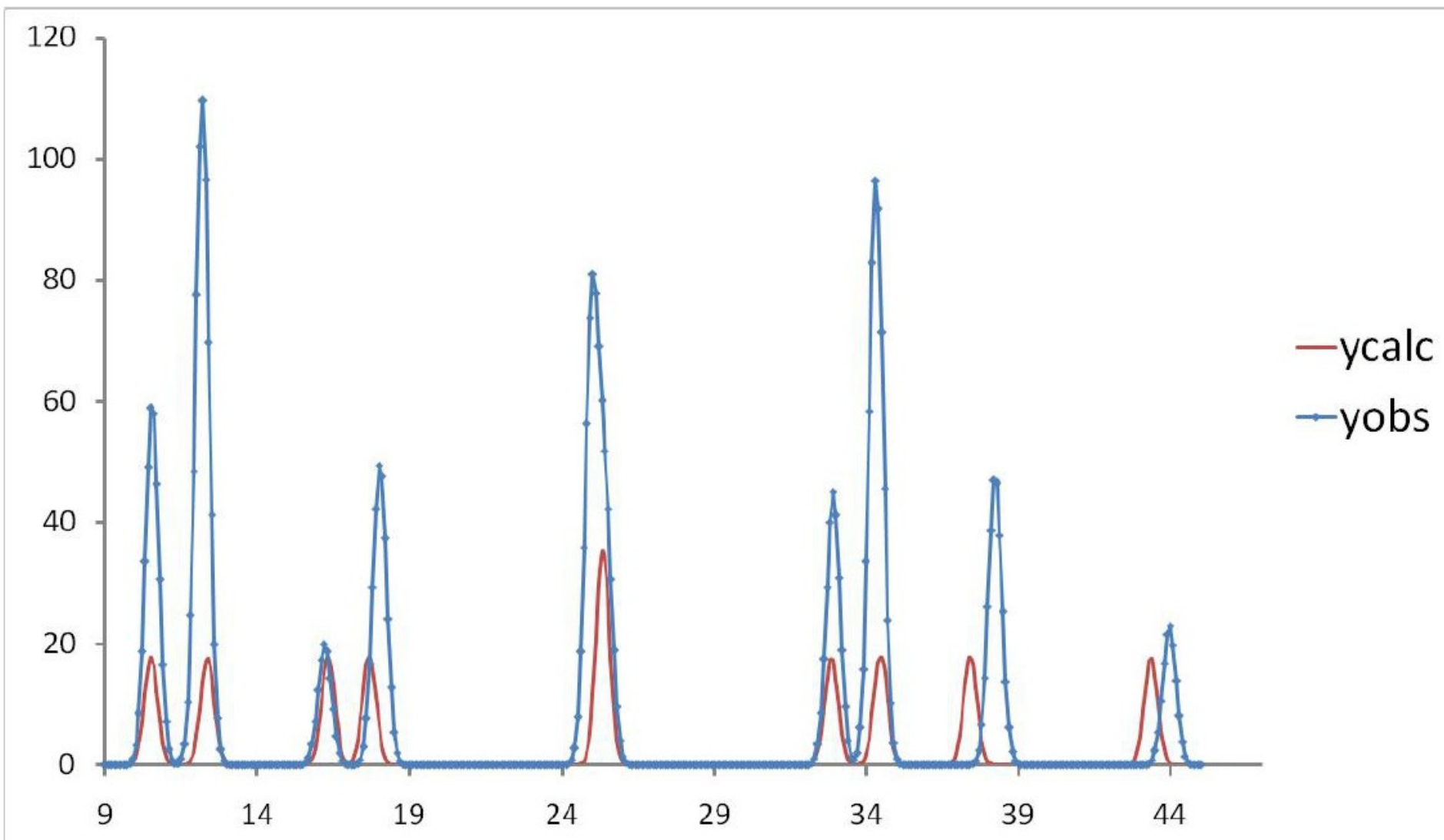
$$I^A = 166$$

$$I^B = 119$$

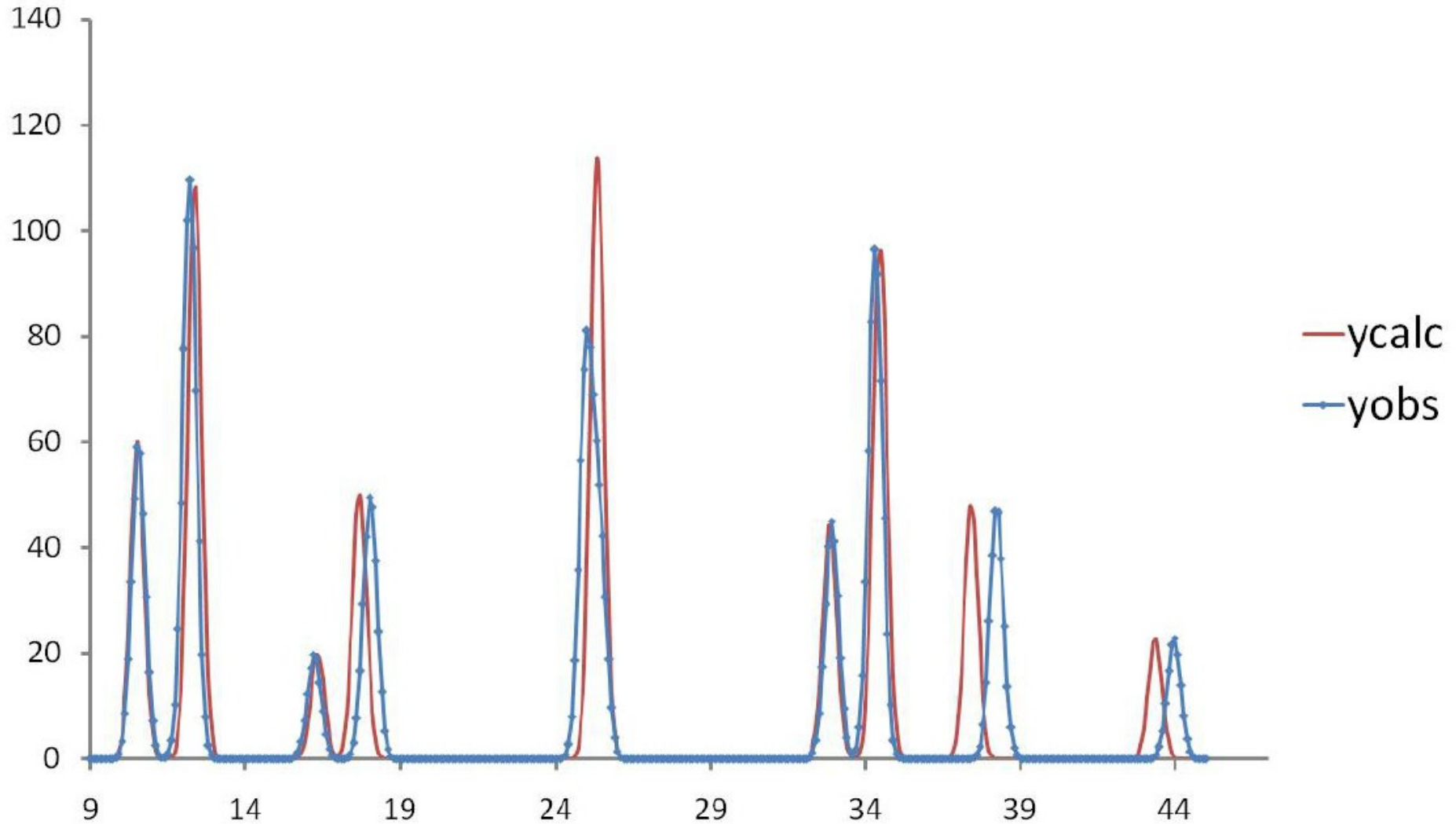




Le Bail method



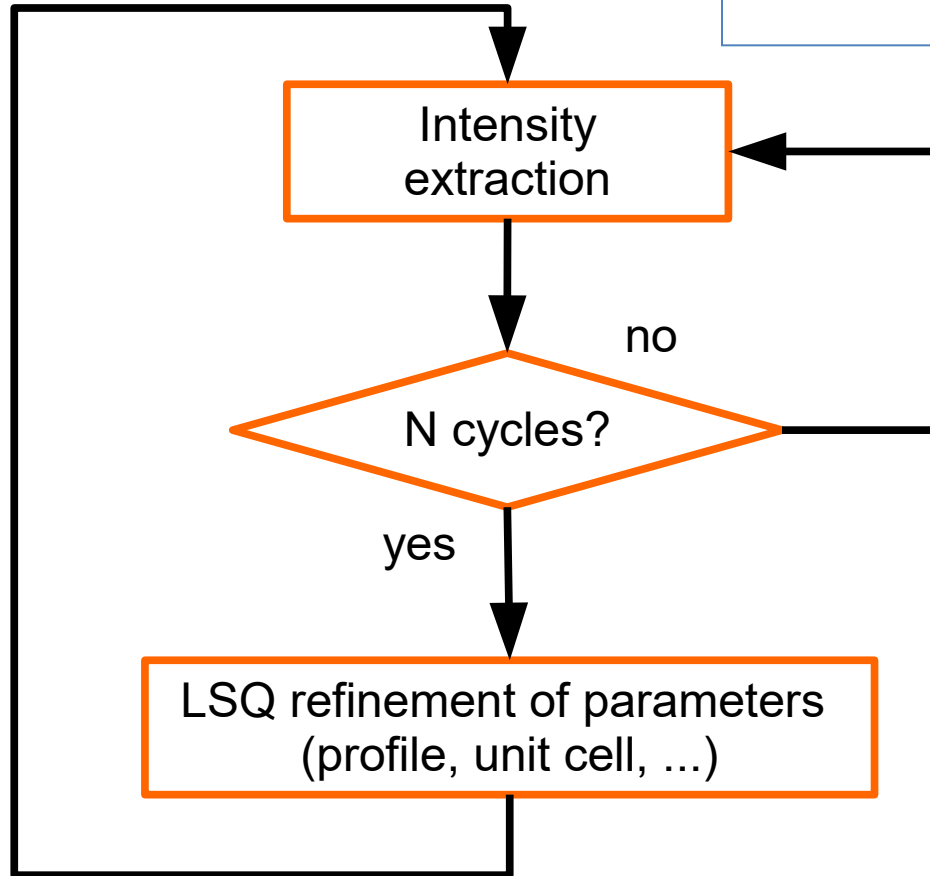
Le Bail method





Le Bail method

$$I^A(obs) = \sum_i y_i(obs) \frac{y_i^A}{y_i^A + y_i^B}$$



Refinement of profile and unit cell parameters by least squares methods combined with the calculation of peak's intensities



Profile parameters

Gaussian:

$$G(b_G, x) = \frac{1}{b_G \sqrt{2\pi}} \exp(-x^2 / 2b_G^2)$$

$$b_G^2 = \frac{H_G^2}{8 \ln 2}$$

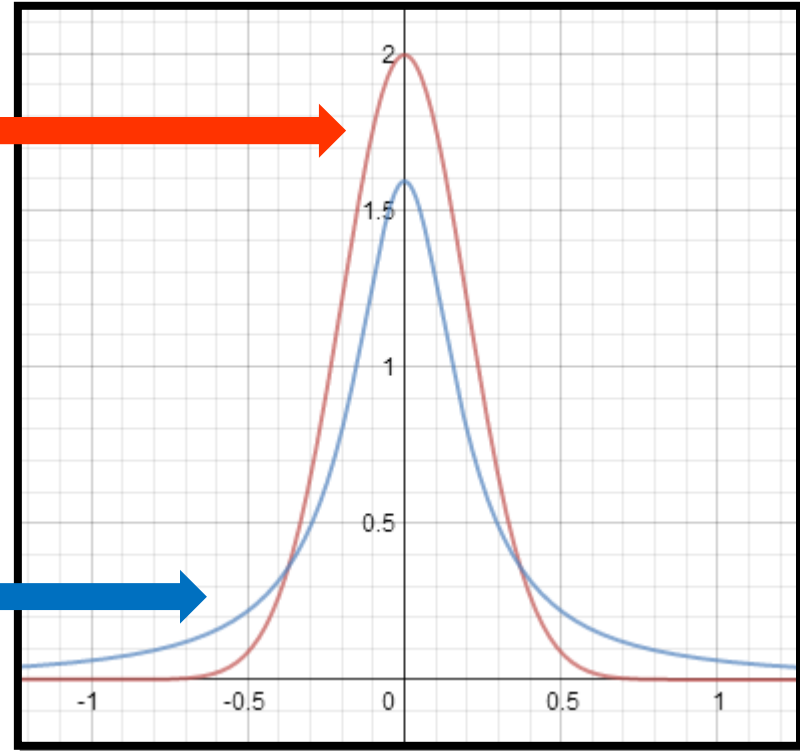
where H_G is „width“ of the peak

Lorentzian:

$$L(b_L, x) = \frac{2}{\pi b_L} \frac{1}{1 + (2x/b_L)^2}$$

$$b_L = H_L$$

where H_L is „width“ of the peak





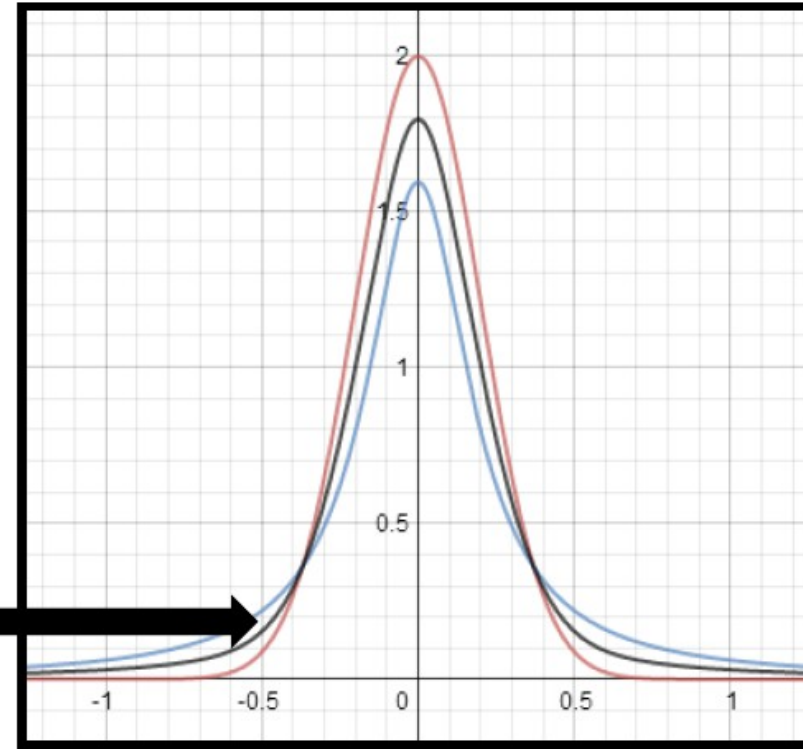
Profile parameters

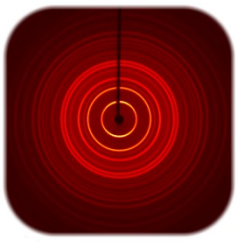
Voigt function:

$$V(b_G, b_L, x) = \int_{-\infty}^{+\infty} G(b_G, x') L(b_L, x - x') dx'$$

It is a combination of **Gaussian** and **Lorentzian**.
For the description of powder profiles, we are using pseudo-voigt function:

$$pV(H, x) = \eta L(H, x) + (1 - \eta) G(H, x)$$





Profile parameters

For gaussian, we are usually using parameters according to Cagliotti, Pauletti & Ricci, 1958 (Nucl.Instrum., **3**, 223):

$$b_G^2 = U \tan^2 \theta + V \tan \theta + W + \frac{P}{\cos^2 \theta}$$

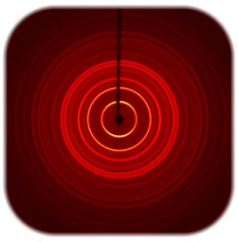
Because there is an equation:

$$\tan^2 \theta + 1 = \frac{1}{\cos^2 \theta}$$

It means, that these parameters U, W, P are strongly correlating

For Lorentzian, we are using:

$$b_L = X_L / \cos \theta + Y_L \tan \theta$$

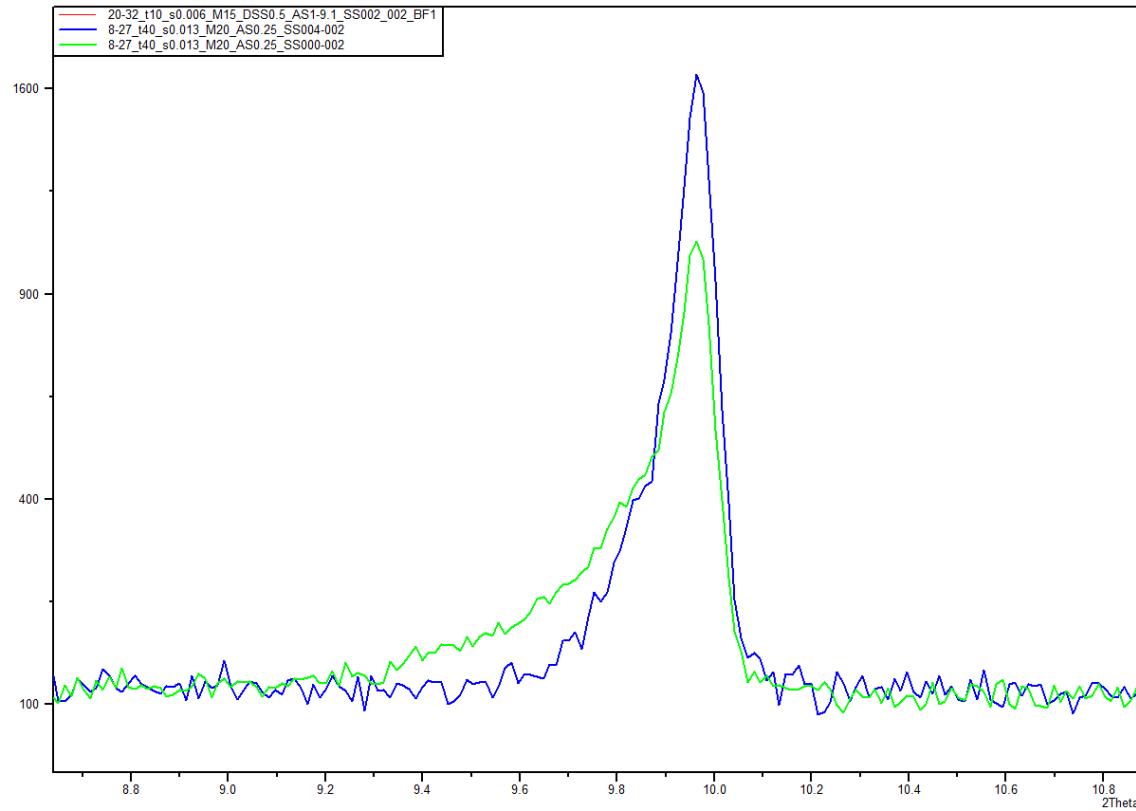


Asymmetry

- Asymmetry is caused by the fact, that **beam source**, **sample** and **detector** have non-zero dimensions.

It is possible to correct it by:

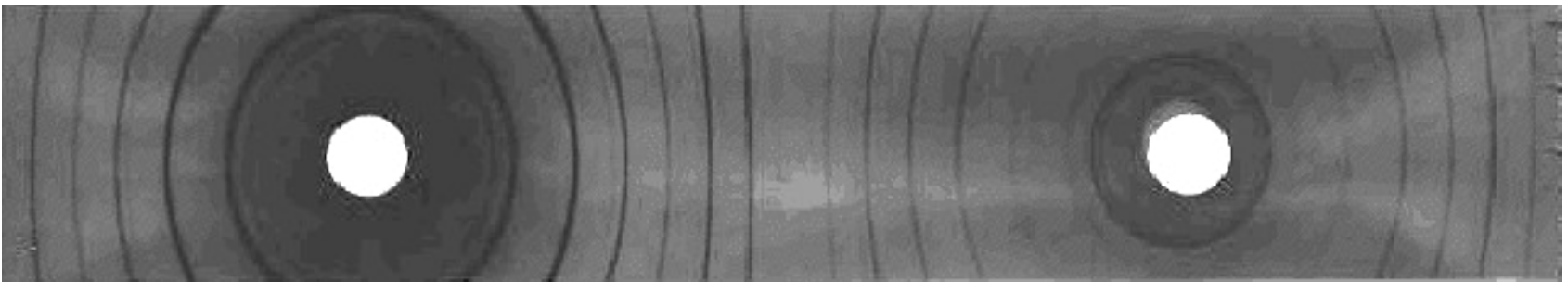
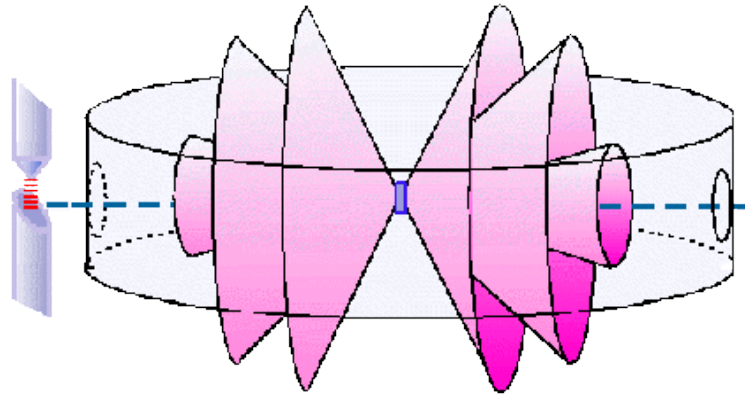
- smaller slits
- soller's slits





Asymmetry

- Influence of the detector's size

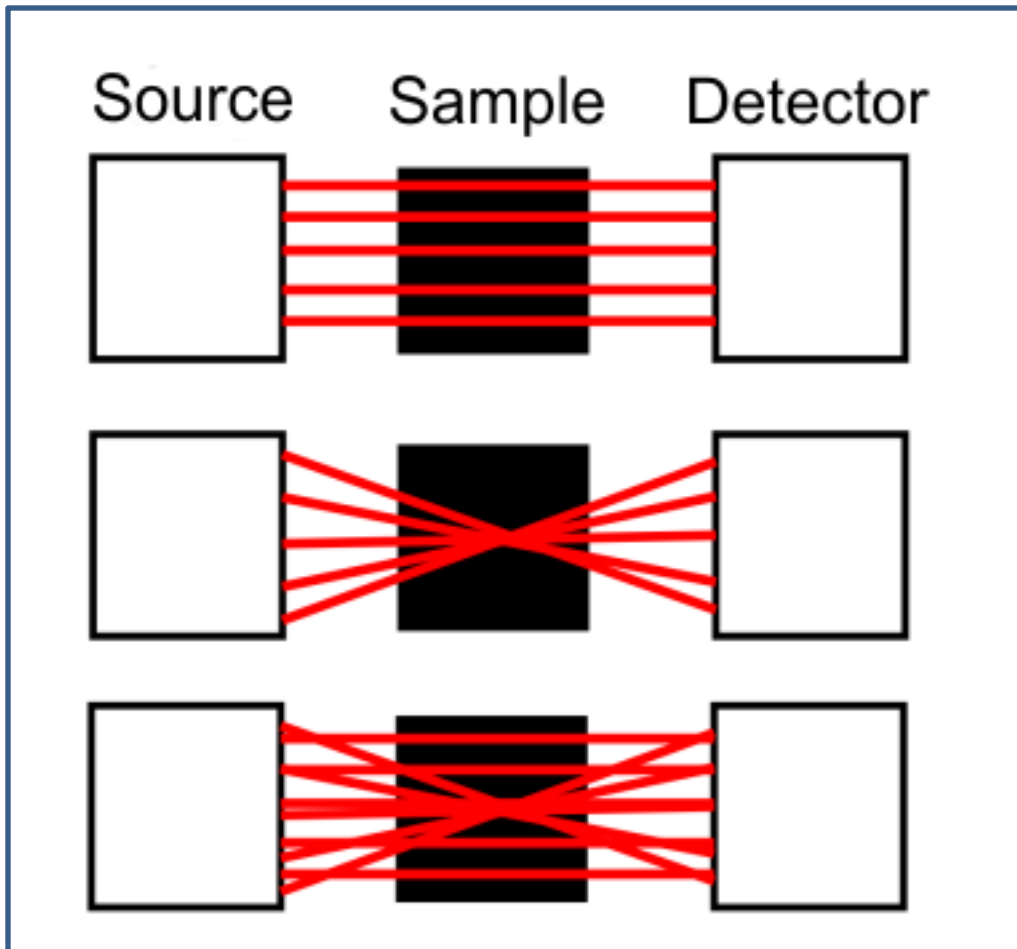




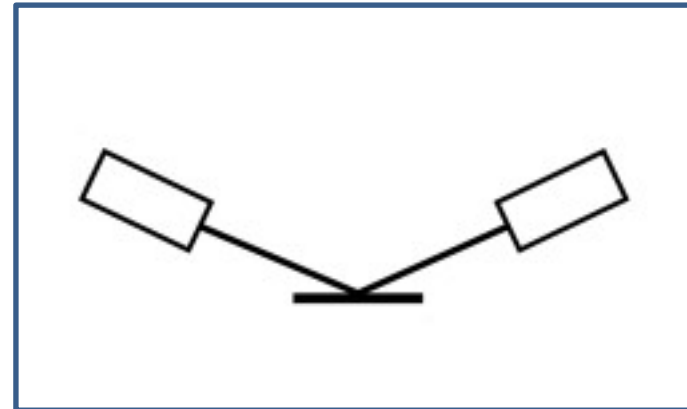
Asymmetry

- Influence of the source and sample size

View from the top



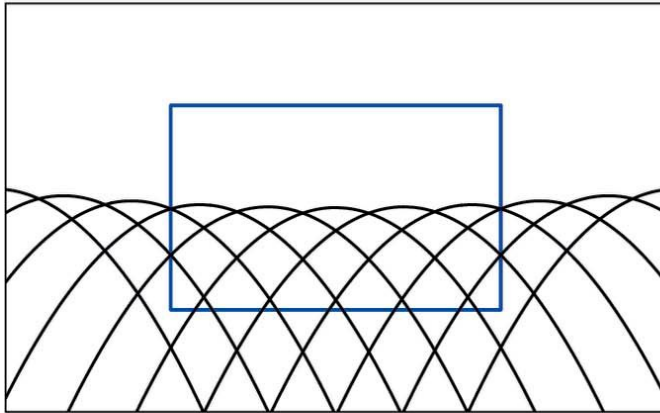
View from side



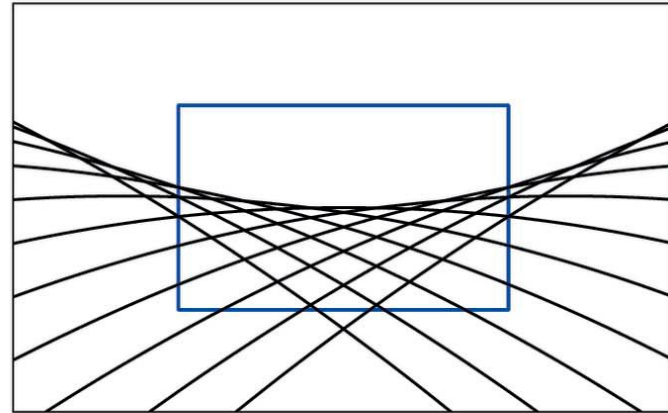


Asymmetry

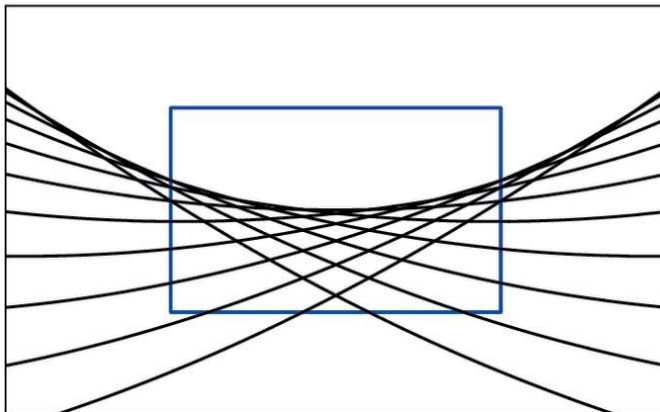
$$2\theta = 20^\circ, \quad 2\varphi = 20^\circ$$



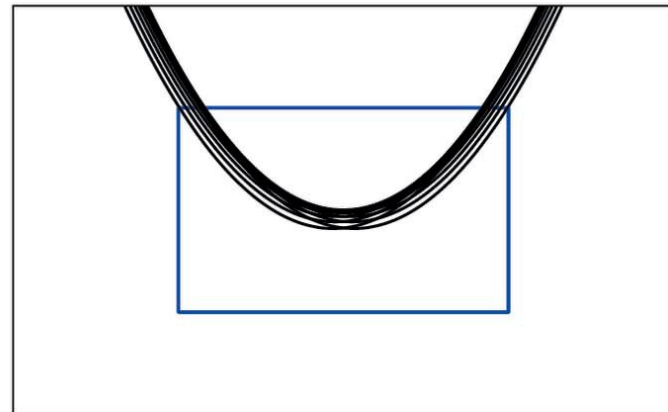
$$2\theta = 80^\circ, \quad 2\varphi = 80^\circ$$



$$2\theta = 100^\circ, \quad 2\varphi = 100^\circ$$

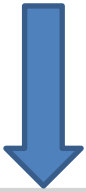
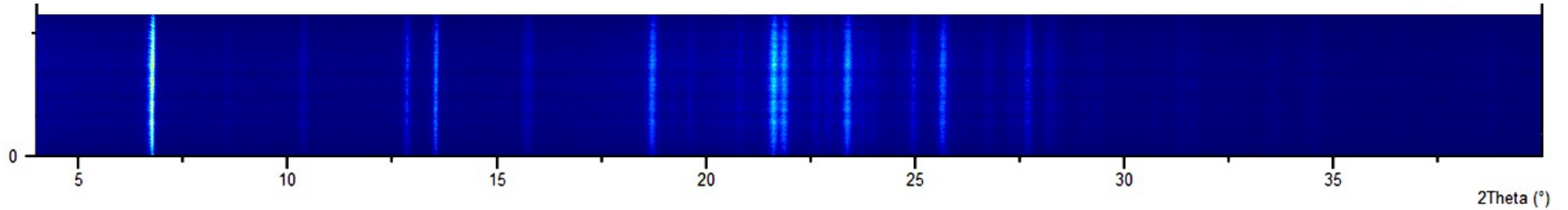


$$2\theta = 160^\circ, \quad 2\varphi = 160^\circ$$

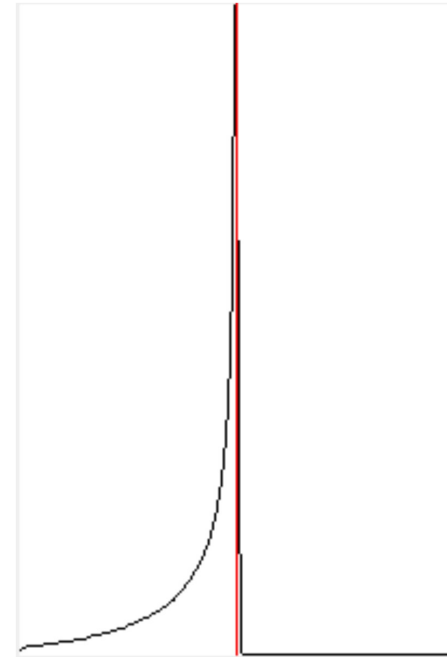
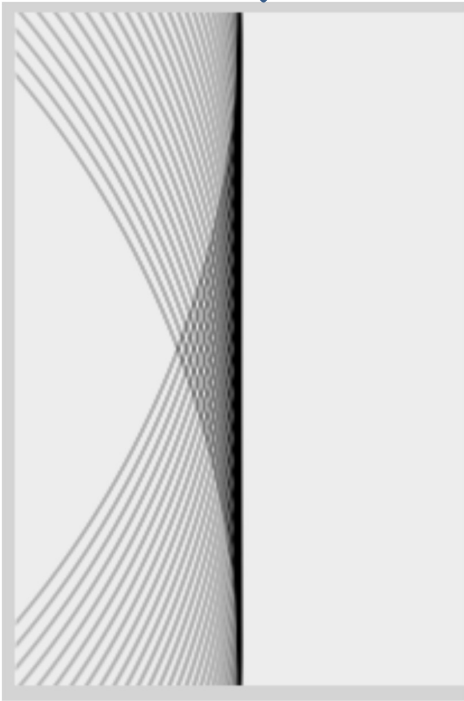




Asymmetry



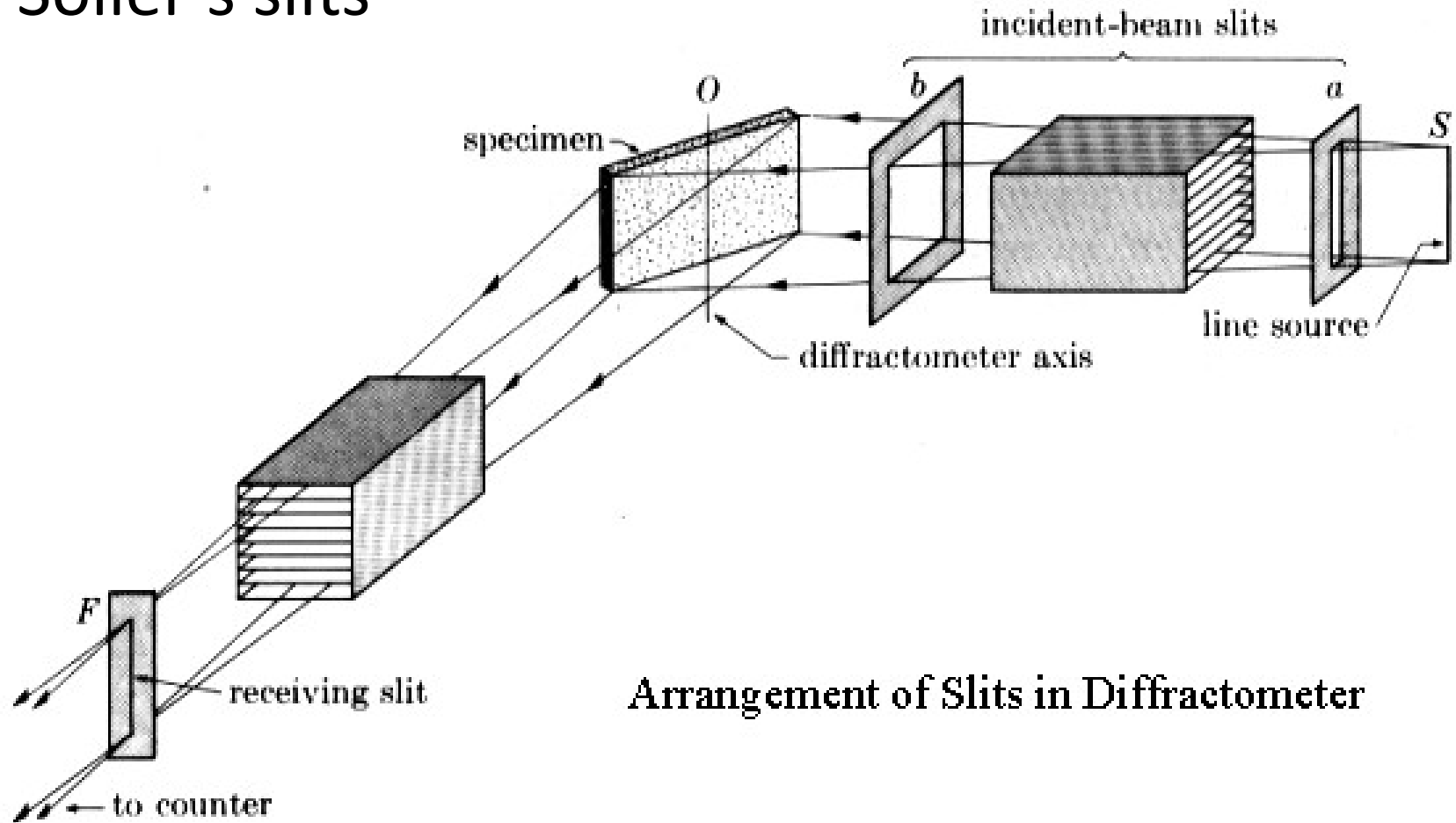
The sample is diffracting as cones...





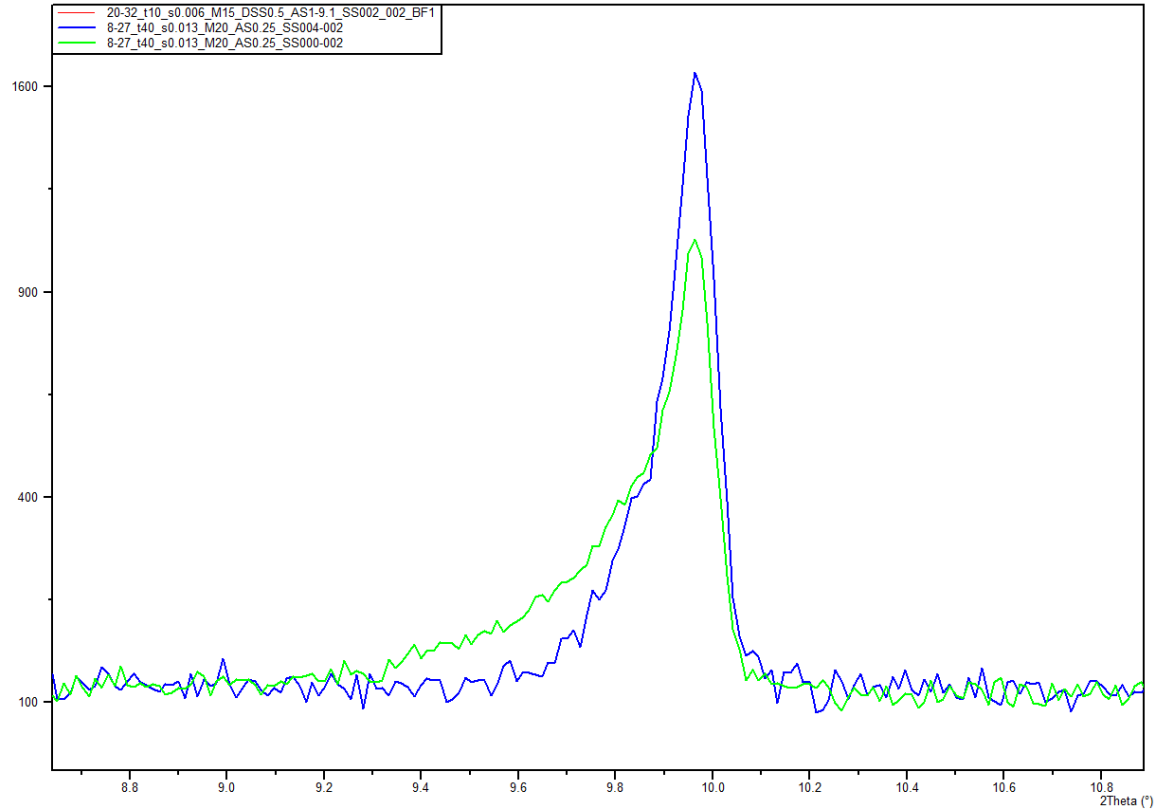
Asymmetry

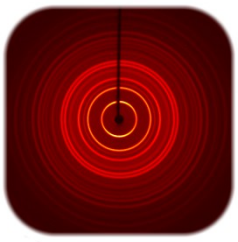
- Soller's slits



Arrangement of Slits in Diffractometer

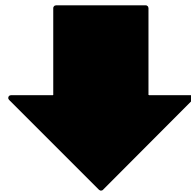
Influence of the soller's slits





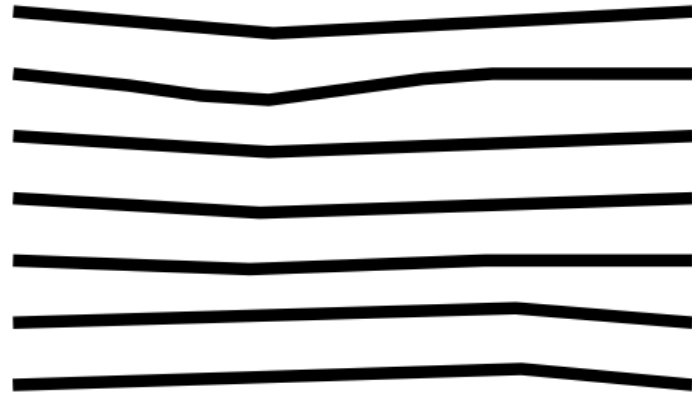
Strain

constant d_{hkl}



Peak width is mainly given by
instrumental function

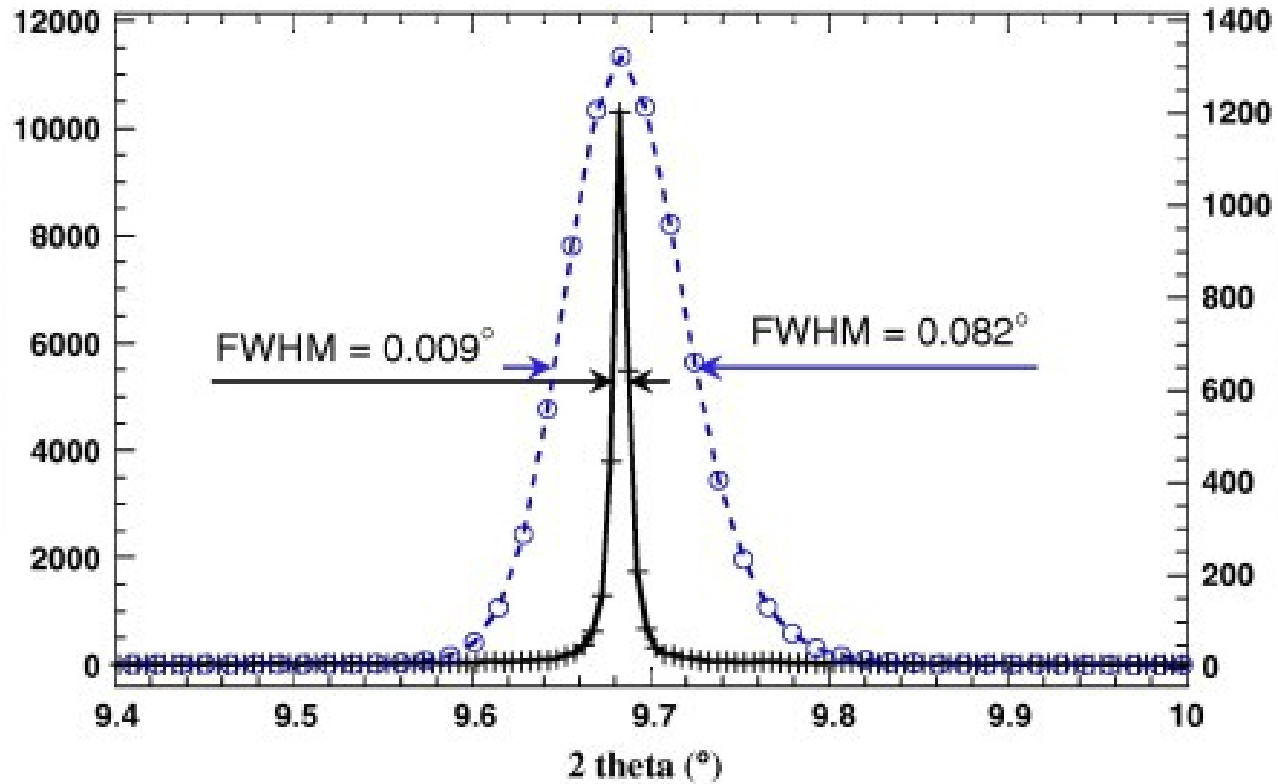
*d_{hkl} is changing from
 $d_{min\ hkl}$ to $d_{max\ hkl}$*



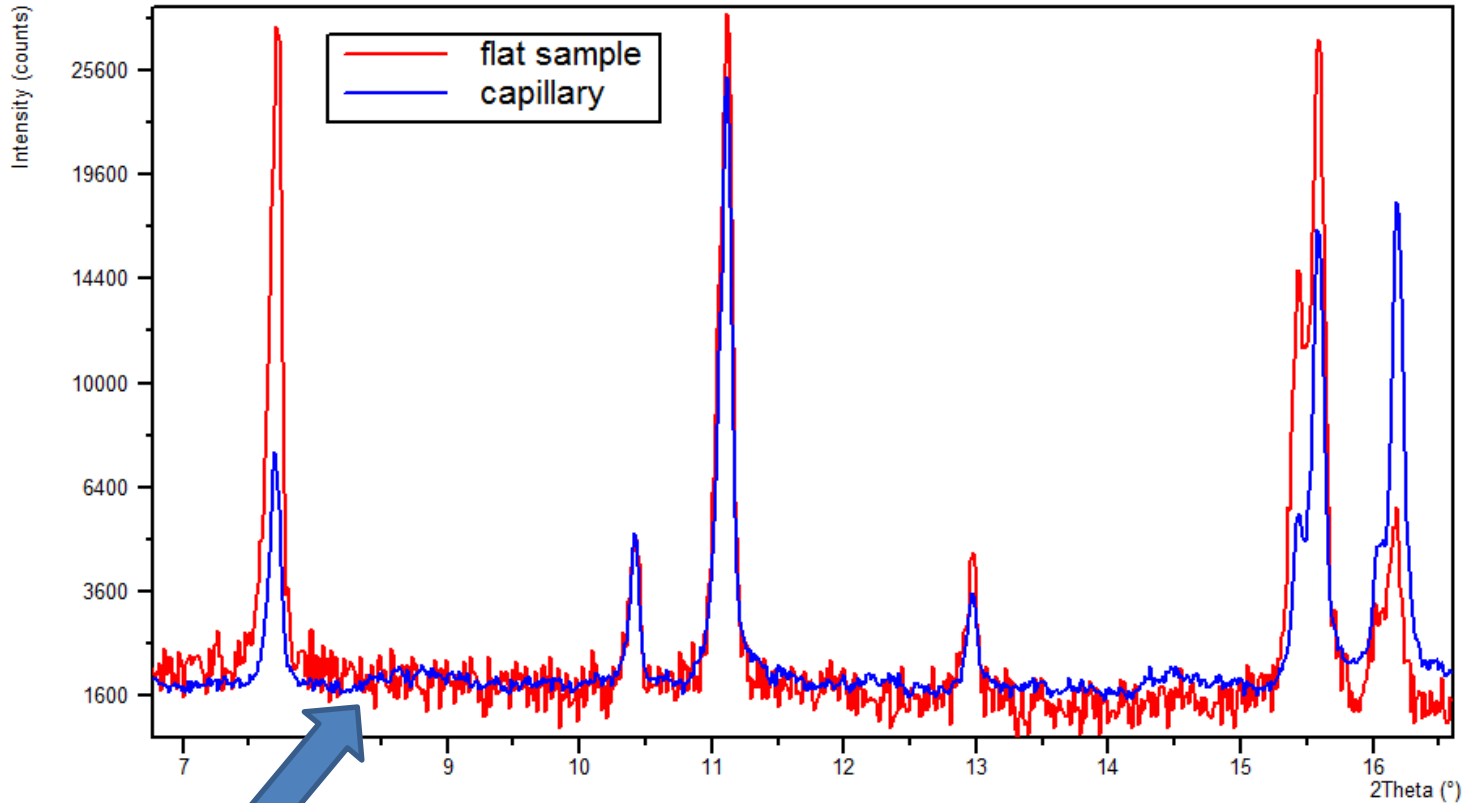
Peak width is given by instrumental
function + distribution of d_{hkl} s in all
single-crystals



Strain



Preferred orientation



March-Dollase coefficient

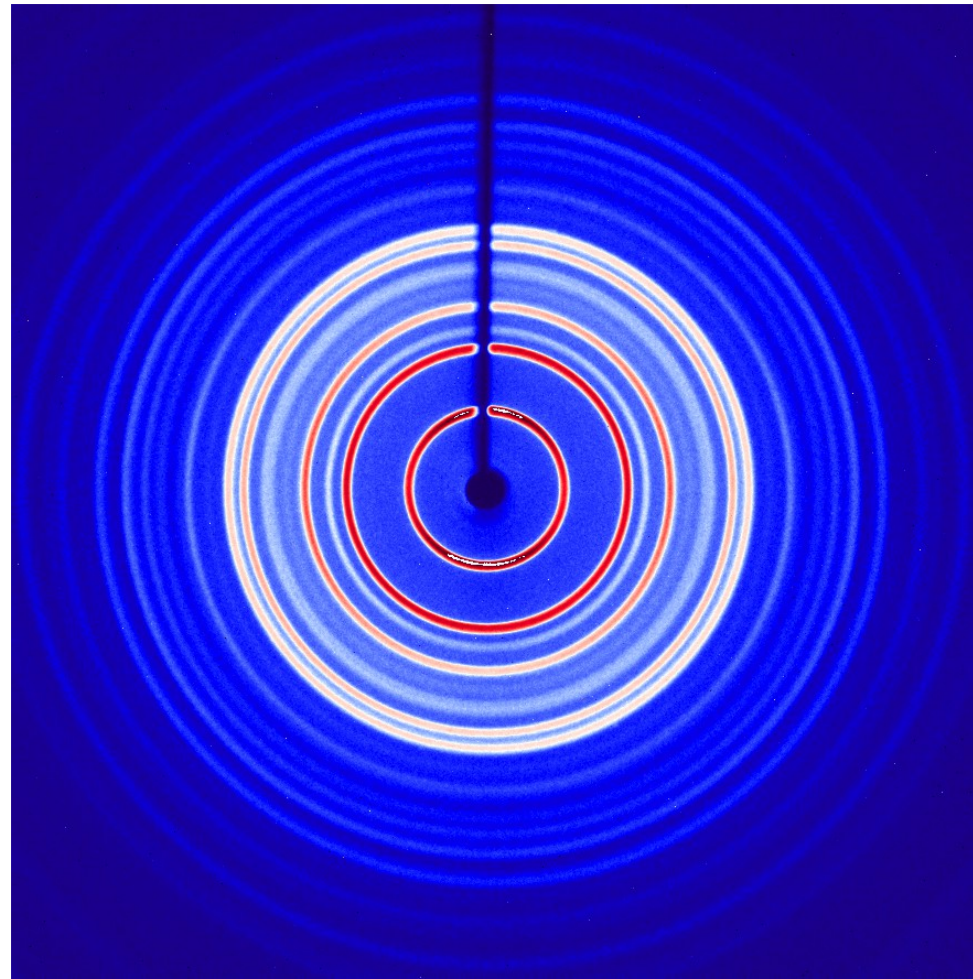
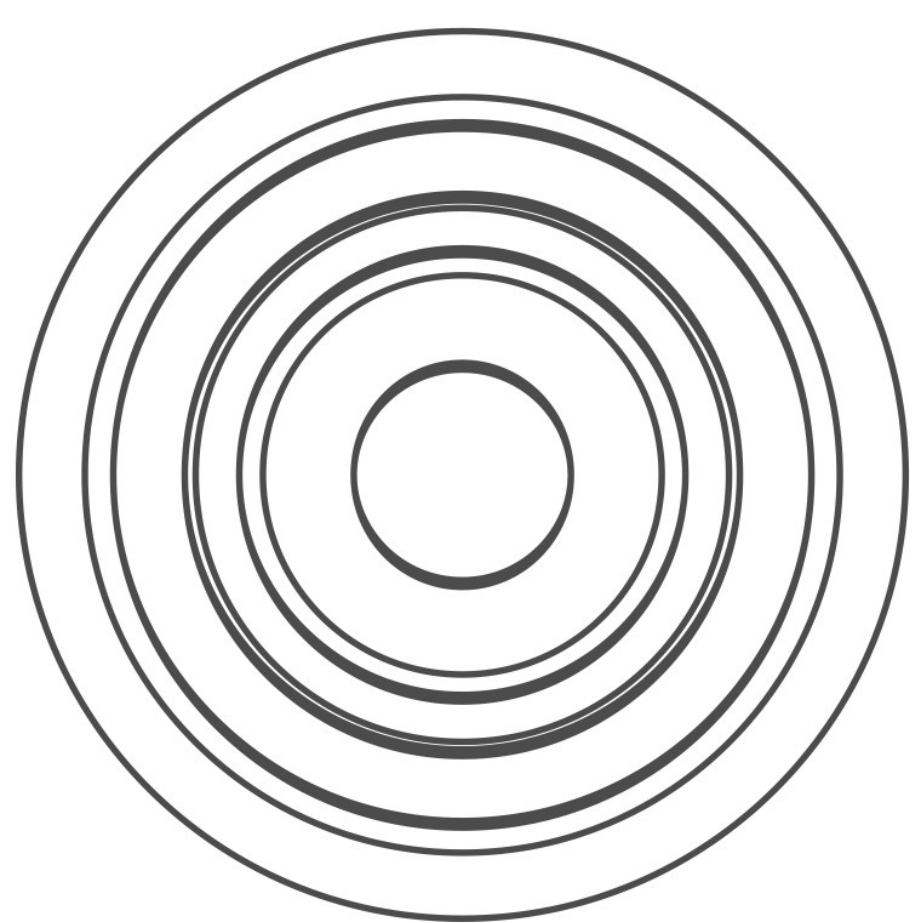
$$O_{ph} = \frac{1}{M_p} \sum_{j=1}^{M_p} \left(R_o^2 \cos^2 A_j + \frac{\sin^2 A_j}{R_o} \right)^{-\frac{3}{2}}$$

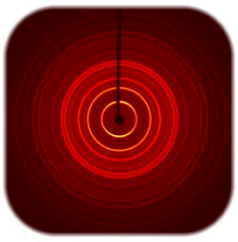
Angle between the direction and current direction

Refined parameter

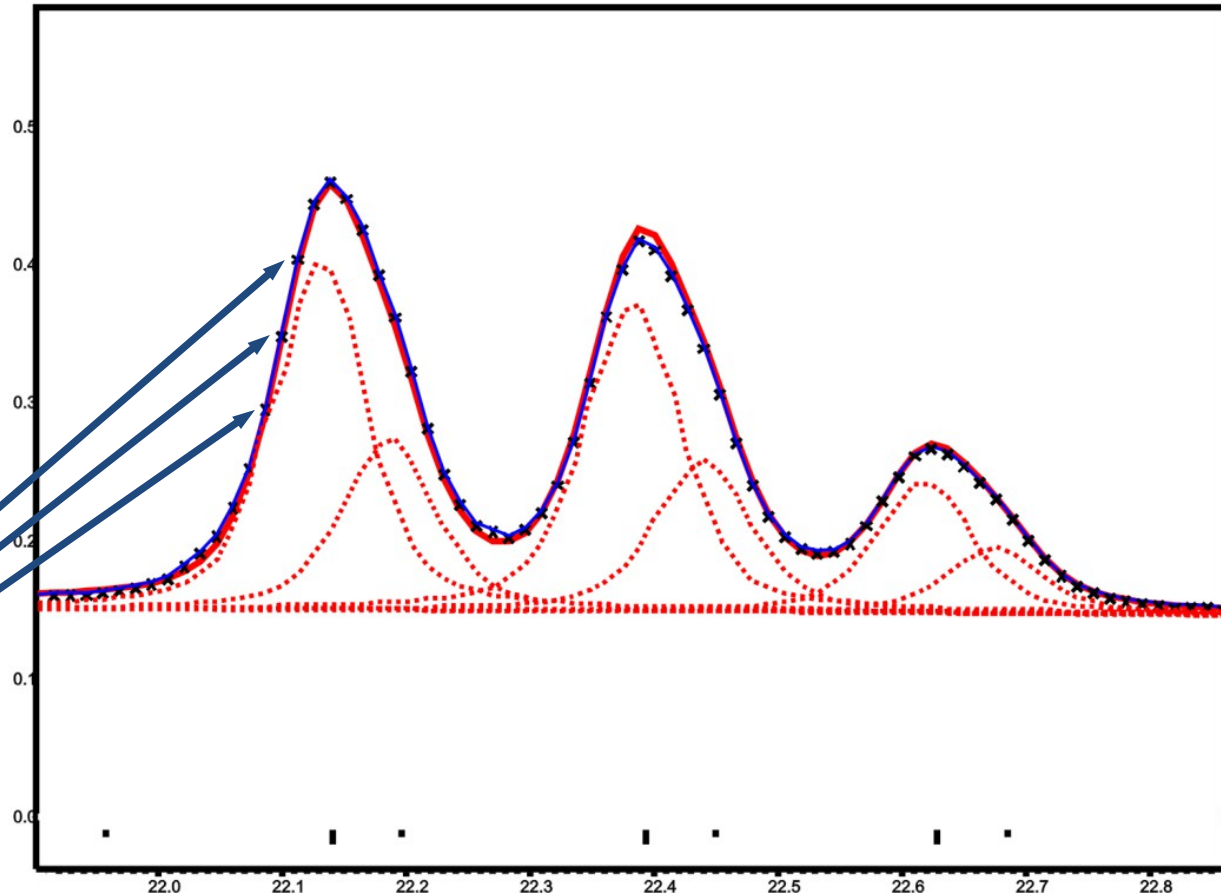


Preferred orientation





Powder pattern – final equation



Intensity at every point

$$y_{ci} = s \sum_h L_h |F_h|^2 \delta(2\theta_i - 2\theta_h) P_h A + y_{bi}$$

s – scale factor, L_h – Lorentz, polarization and multiplicity factor, F_h – structure factor, δ – reflection profile, P_h – preferred orientation function, A – absorption factor, y_{bi} - background



End

Thank you !